

Multivariable analysis in clinical epidemiology

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Why do we need multivariable analysis?

“Treatment (control)” for the confounding effects at analytical level

- ✓ Stratification by confounder(s)
- ✓ **Multivariable / multiple** analysis

Prediction of individual risk



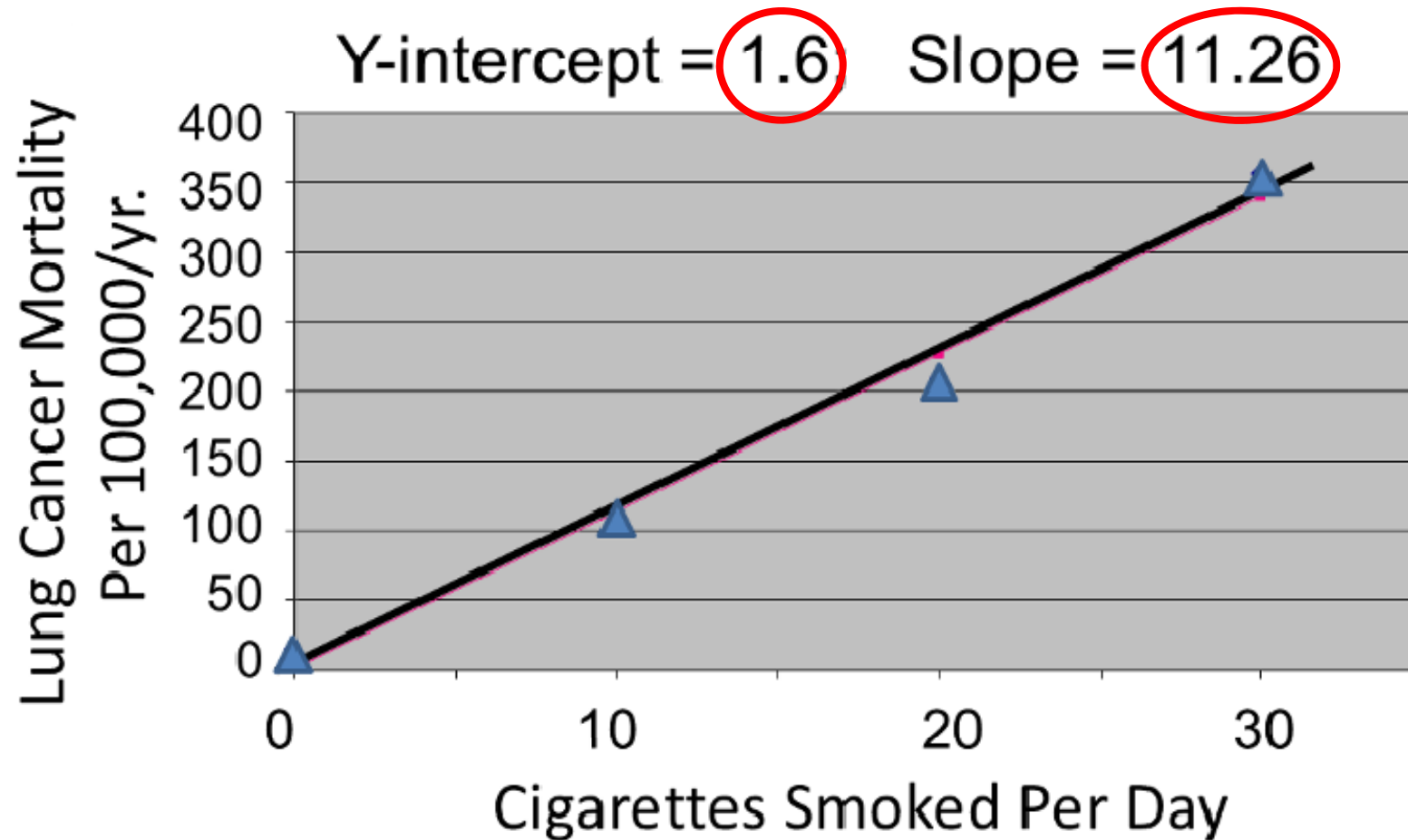
Regression models for multivariable analysis

Paired?	Outcome variable	Proper model
No	Continuous	Linear regression model
	Binomial	Logistic regression model
	Categorical (≥ 3)	Multinomial (polytomous) logistic regression model
	Binomial (event) with censoring	Cox proportional hazard model
Yes	Continuous	Mixed effect model, Generalized estimating equation
	Categorical (≥ 3)	Generalized estimating equation



LINEAR REGRESSION ANALYSIS

Lung cancer mortality by daily cigarettes smoked



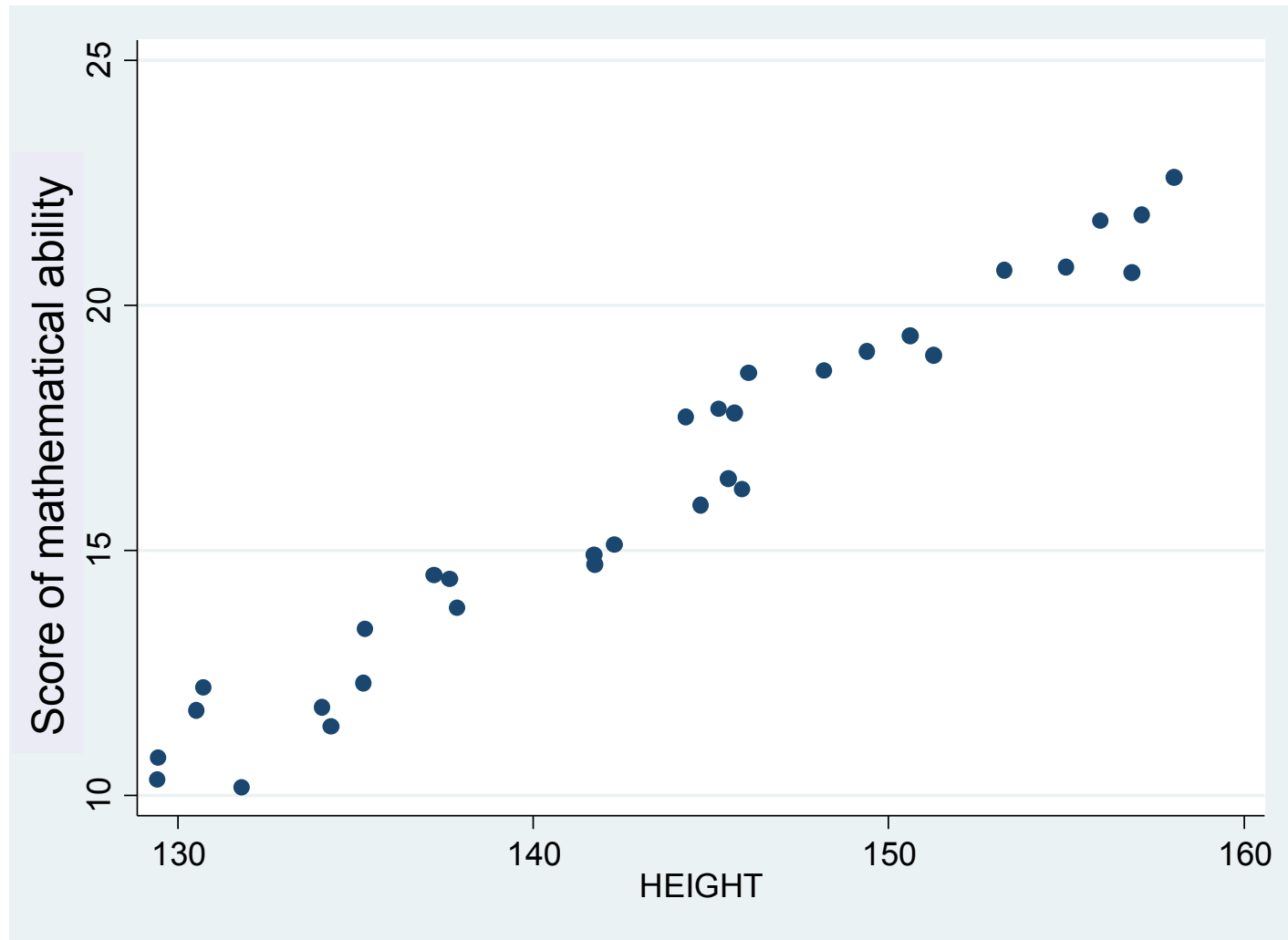
Original data: Doll and Hill Br Med J 1956



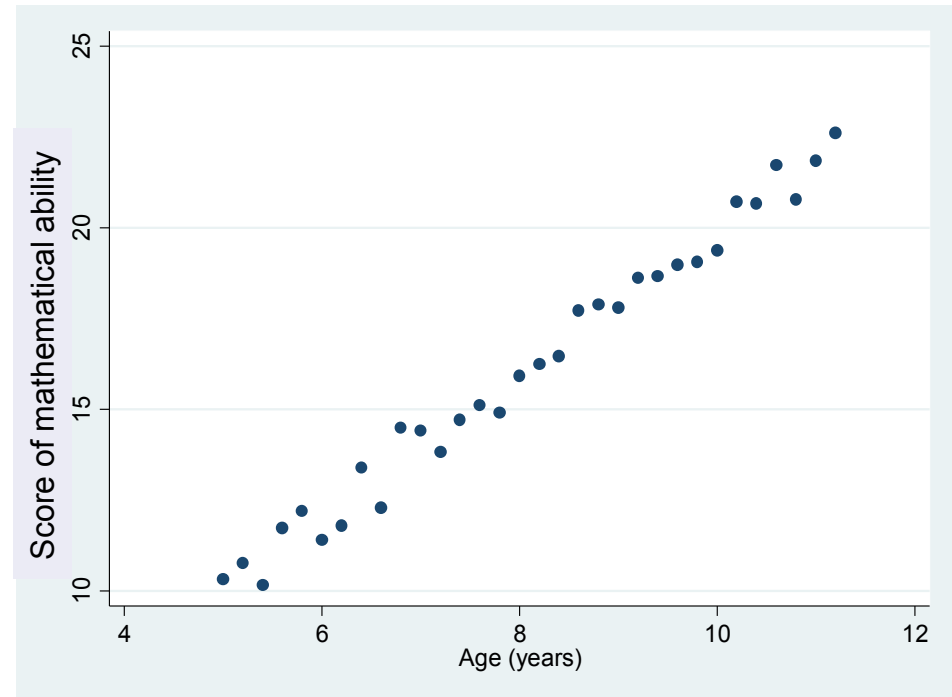
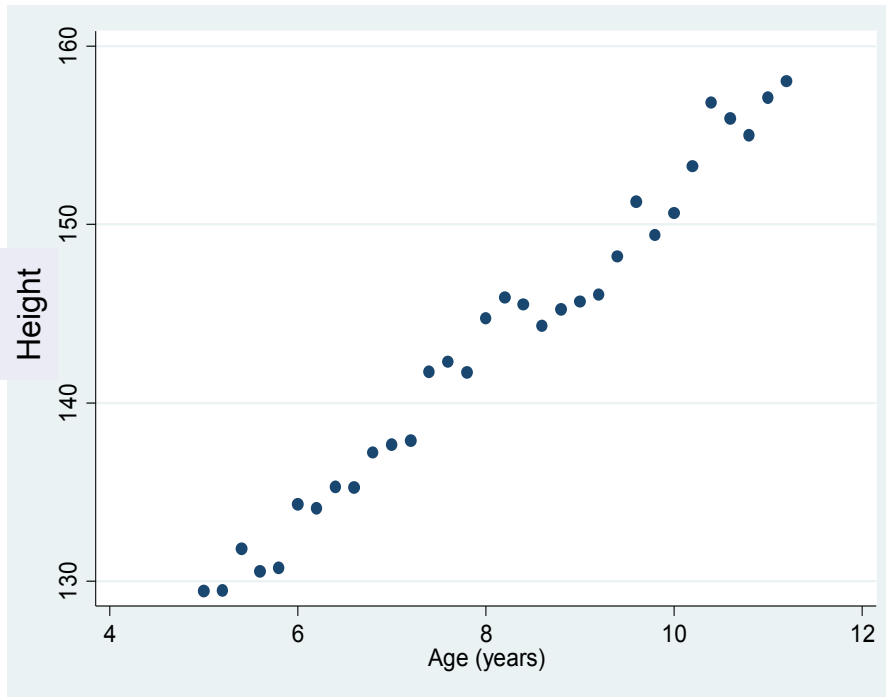
Number of obs = 32
F(1, 30) = 726.87
Prob > F = 0.0000
R-squared = 0.9604
Adj R-squared = 0.9590
Root MSE = .75358

	ama	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
height		.4118029	.0152743	26.96	0.000	.3806086 .4429973
_cons		-42.82525	2.191352	-19.54	0.000	-47.30059 -38.34992

Association between height and score of maths



Both height and ability of maths increase with age



Age is a **confounding factor** in the association between height and ability of maths.





How age itself influences the association between height and the ability of maths?

Let's see the equation

$$\text{Ability of maths (AM)} = \alpha + \beta_1(\text{Height})$$

$$\rightarrow \text{AM} = -42.8 + 0.41(\text{Height})$$

$$\text{AM} = \alpha + \beta_1(\text{Height}) + \beta_2(\text{Age})$$

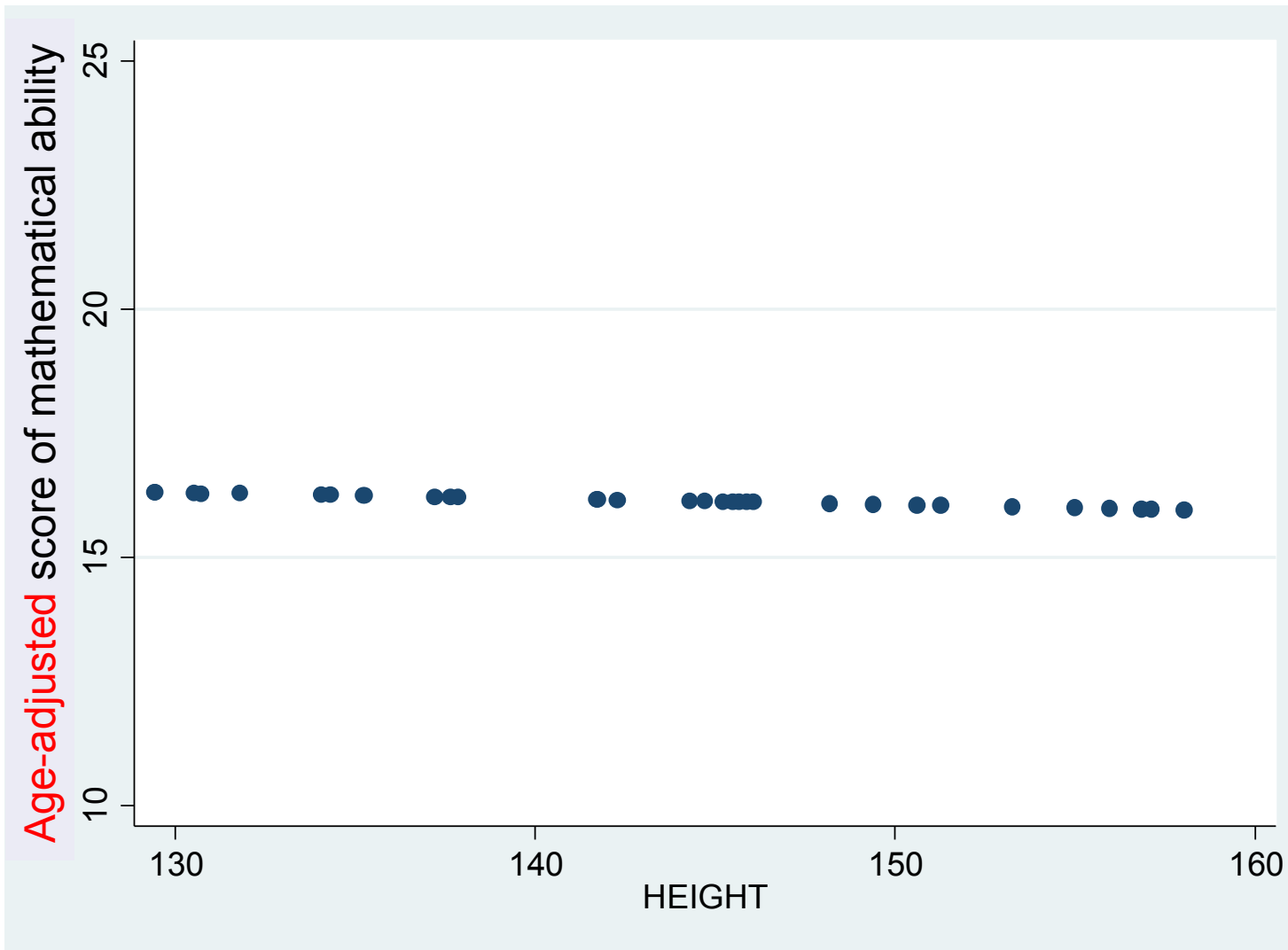
$$\rightarrow \text{AM} = 1.48 - 0.01(\text{Height}) + 2.02(\text{Age})$$

Significant association between height and the ability of maths was gone after adjusting for the effect of age

Source	SS	df	MS	Number of obs =	32
-----+-----				F(2, 29) =	851.23
Model	422.6119	2	211.305972	Prob > F =	0.0000
Residual	7.19885	29	.248236138	R-squared =	0.9833
-----+-----				Adj R-squared =	0.9821
Total	429.81079	31	13.8648643	Root MSE =	.49823

ama	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----						
height 	-.0121303	.0680948	-0.18	0.860	-.1513998	.1271393
age	2.02461	.3216095	6.30	0.000	1.366845	2.682375
_cons	1.483038	7.185946	0.21	0.838	-13.21387	16.17995
-----+-----						

No association between height and score of maths



Interpretation of coefficients

Let's see the equation

$$\text{Ability of maths (AM)} = \alpha + \beta_1(\text{Height})$$

$$\rightarrow \text{AM} = -42.8 + 0.41(\text{Height})$$

0.41 points increase by 1cm increase of height

$$\text{AM} = \alpha + \beta_1(\text{Height}) + \beta_2(\text{Age})$$

$$\rightarrow \text{AM} = 1.48 - 0.01(\text{Height}) + 2.02(\text{Age})$$

0.01 points decrease by 1cm increase of height

Confounding effect: magnitude and direction of the association



Interpretation of coefficients in general

To simplify, the explanatory variable is binomial one: 1=exposed or 0=unexposed

Exposed: $Y_e = \alpha + \beta(\text{Exp}=1) = \alpha + \beta$

Unexposed: $Y_u = \alpha + \beta(\text{Exp}=0) = \alpha$

Difference: $Y_e - Y_u = \beta$

■ **Coefficient estimate: difference in dependent value**



Interpretation of coefficients after **log-transformation** of dependent variable

The explanatory variable is binomial one:
1=exposed or 0=unexposed

Exposed: $\ln(Y_e) = \alpha + \beta(\text{Exp}=1) = \alpha + \beta$

Unexposed: $\ln(Y_u) = \alpha + \beta(\text{Exp}=0) = \alpha$

Difference: $\ln(Y_e) - \ln(Y_u) = \beta$

Ratio: $Y_e / Y_u = e^{\beta}$

- **Coefficient estimate: ratio of dependent value (after exponentiating)**



LOGISTIC REGRESSION ANALYSIS

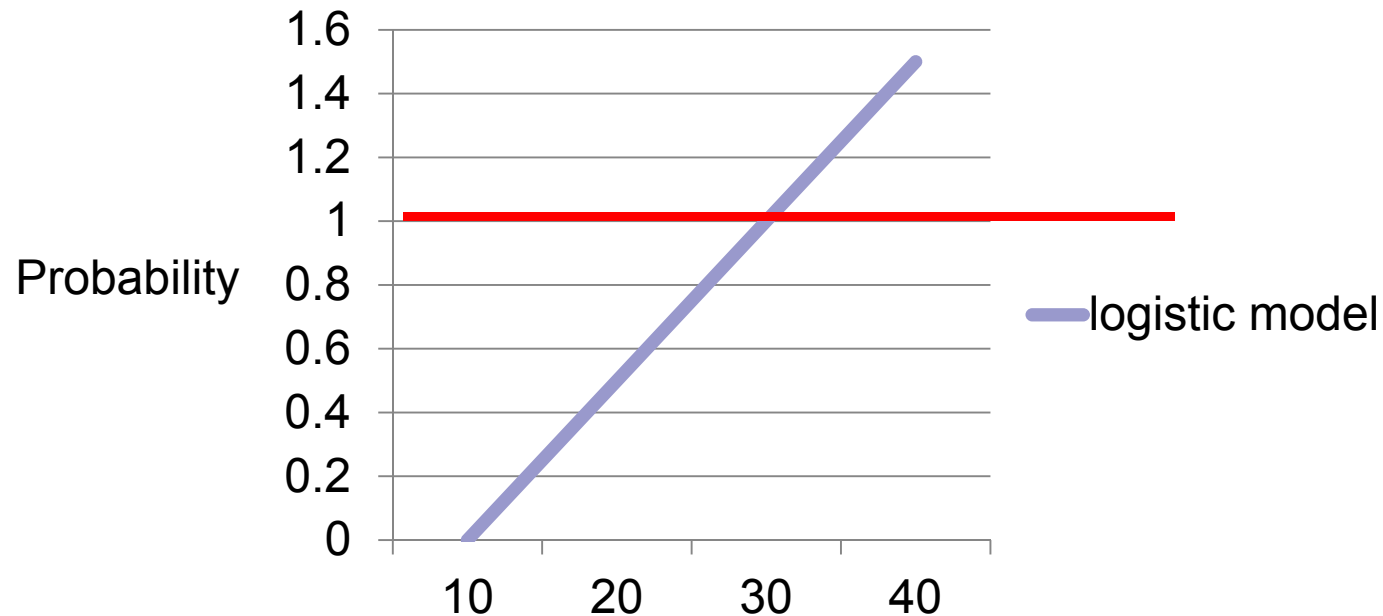


Logistic regression analysis

- Logistic regression is used to model the probability of a binary response as a function of a set of variables thought to possibly affect the response (called covariates).

$Y = \begin{cases} 1: \text{case (with the disease)} \\ 0: \text{control (no disease)} \end{cases}$

One could imagine trying to fit a linear model (since this is the simplest model !) for the probabilities, but often this leads to problems:



In a linear model, fitted probabilities can fall outside of 0 to 1. Because of this, linear models are seldom used to fit probabilities.



In a logistic regression analysis, the **logit** of the probability is modeled, rather than the probability itself.

P = probability of getting disease (0 ~ 1)

$$\text{logit}(p) = \text{log} \left[\frac{p}{1-p} \right]$$

This transformation allows us to use a linear model.

As always, we use the natural log.

The logit is therefore **the log odds**, since $\text{odds} = p / (1-p)$

Logistic regression model

Now, we have the same function with linear regression model in the right side.

$$\text{logit}(p_x) = \log \left[\frac{p_x}{1 - p_x} \right] = \alpha + \beta x$$

where p_x = probability of event for a given value x , and α and β are unknown parameters to be estimated from the data.

→ **Multivariable analysis** is applicable to adjust the effect of confounding factor.

Interpretation of coefficients of logistic regression model

The explanatory variable is binomial one:
1=exposed or 0=unexposed

Exposed: $\log(O_e) = \alpha + \beta(\text{Exp}=1) = \alpha + \beta$

Unexposed: $\log(O_u) = \alpha + \beta(\text{Exp}=0) = \alpha$

Difference: $\log(O_e) - \log(O_u) = \beta$

Odds ratio: $O_e / O_u = e^{\beta}$

■ **Coefficient estimate: Odds ratio (after exponentiating)**



STRATEGY FOR CONSTRUCTING REGRESSION MODELS



Basic principles

1. **Stratified analysis** should be first.
2. Determine which **confounders to include** in the model. 
3. Estimate the shape of the exposure-disease relation.

Dose-response relation

1. Evaluate **interaction**

How to determine confounders: data-dependent manner



1. Start with a set of predictors of outcome based on the strength of their relation to the outcome.
2. Build a model by introducing predictor variables one at a time: check the amount of change in the coefficient of the exposure term
> 10% change: include it as a confounder



Example of a confounder (age)

Ability of maths (AM) = $\alpha + \beta_1(\text{Height})$

→ AM = $-42.8 + 0.41(\text{Height})$

AM = $\alpha + \beta_1(\text{Height}) + \beta_2(\text{Age})$

→ AM = $1.48 - 0.01(\text{Height}) + 2.02(\text{Age})$

How to determine confounders: data-independent manner



Some researchers argue that
“Without data analysis, decide
confounders, important risk factors of
the outcome, based on the previous
studies.”

**How can we pick-up “important risk factors”?
If there are few studies, how can we know
confounders?**



How many explanatory variables can we use in a model?

Model	Number of explanatory variables	Example
Linear regression model	Sample size / 15	<u>Up to around 6-7 variables</u> in 100 subjects
Logistic regression model	Smaller sample size of outcome / 10	<u>Up to 10 variables</u> if the numbers of cases and controls are 100 and 300, respectively.
Cox proportional hazard model	The number of event / 10	<u>Up to 9 variables</u> if you have 90 events out of 150 subjects



ATTENTION!

- When you include a categorical variable in your model, you have to count that as “the number of categories – 1”.
- For example, the variable of age group used in the previous practice, we have to count it as “two” (=3 categories -1) variables.

If you cannot recruit enough sample size

- Calculate “**propensity score**” which can be used for adjustment of confounding effects.

Example

Aspirin Use and All-Cause Mortality Among Patients Being Evaluated for Known or Suspected Coronary Artery Disease

A Propensity Analysis

Patricia A. Gum, MD

Maran Thamilarasan, MD

Junko Watanabe, MD

Eugene H. Blackstone, MD

Michael S. Lauer, MD

Context Although aspirin has been shown to reduce cardiovascular morbidity and short-term mortality following acute myocardial infarction, the association between its use and long-term all-cause mortality has not been well defined.

Objectives To determine whether aspirin is associated with a mortality benefit in stable patients with known or suspected coronary disease and to identify patient characteristics that predict the maximum absolute mortality benefit from aspirin.

Table 1. Baseline and Exercise Characteristics According to Aspirin Use*

Variable	Aspirin (n = 2310)	No Aspirin (n = 3864)	P Value
Demographics			
Age, mean (SD), y	62 (11)	56 (12)	<.001
Men, No. (%)	2167 (94)	2167 (56)	<.001
Clinical history			
Diabetes, No. (%)	432 (19)	432 (11)	<.001
Hypertension, No. (%)	1569 (68)	1569 (41)	<.001
Tobacco use, No. (%)	500 (22)	500 (13)	.001
Prior coronary artery disease, No. (%)	178 (8)	178 (20)	<.001
Prior coronary artery bypass grafting, No. (%)	21 (1)	21 (5)	<.001
Prior percutaneous coronary intervention, No. (%)	667 (29)	148 (4)	<.001
Prior Q-wave MI, No. (%)	369 (16)	285 (7)	<.001
Atrial fibrillation, No. (%)	27 (1)	55 (1)	.04
Congestive heart failure, No. (%)	127 (6)	178 (5)	.12
Medication use			
Digoxin use, No. (%)	171 (7)	216 (6)	.004
β -Blocker use, No. (%)	811 (35)	550 (14)	<.001
Diltiazem/verapamil use, No. (%)	452 (20)	405 (10)	<.001
Nifedipine use, No. (%)	261 (11)	283 (7)	<.001
Lipid-lowering therapy, No. (%)	775 (34)	380 (10)	<.001
ACE inhibitor use, No. (%)	349 (15)	441 (11)	<.001
Cardiovascular assessment and exercise capacity			
Body mass index, mean (SD), kg/m ²	29 (5)	30 (7)	<.001
Ejection fraction, mean (SD), %	50 (9)	53 (7)	<.001
Resting heart rate, mean (SD), beats/min	74 (13)	79 (14)	<.001
Resting blood pressure, mean (SD), mm Hg	130 (14)	130 (14)	.92

Almost all prognostic factors (n=28) are related to aspirin use!

After **matching by propensity score**, the distribution of prognostic factors are similar between aspirin users and non-users.

Table 3. Selected Baseline and Exercise Characteristics According to Aspirin Use In Propensity Matched Cohort

	Aspirin (n = 1351)	No Aspirin (n = 1351)	P Value
Demographics			
Age, mean (SD), y	60 (11)	61 (11)	.16
Men, No. (%)	951 (70)	974 (72)	.33
Clinical history			
Diabetes, No. (%)	203 (15)	207 (15)	.83
Hypertension, No. (%)	679 (50)	698 (52)	.46
Tobacco use, No. (%)	161 (12)	162 (12)	.95
Cardiac variables			
Prior coronary artery disease, No. (%)	652 (48)	659 (49)	.79
Prior coronary artery bypass graft, No. (%)	251 (19)	235 (17)	.42
Prior percutaneous coronary intervention, No. (%)	166 (12)	147 (11)	.25
Prior Q-wave MI, No. (%)	194 (14)	206 (15)	.52
Atrial fibrillation, No. (%)	21 (2)	24 (2)	.65
Congestive heart failure, No. (%)	79 (6)	89 (7)	.43

It is just like a RCT!
(pseud RCT)

Table 4. Cox Proportional Hazards Analyses of Aspirin Use and Mortality Among Propensity-Matched Patients (n = 2702)*

Model	Hazard Ratio (95% CI)	P Value
Unadjusted	0.53 (0.38-0.74)	.002
Adjusted for propensity	0.53 (0.38-0.74)	<.001
Adjusted for propensity and selected variables†	0.59 (0.42-0.83)	.002
Adjusted for propensity and all covariates‡	0.56 (0.40-0.78)	<.001

*CI indicates confidence interval.

†Selected variables included prior coronary artery disease, prior coronary artery bypass grafting, prior percutaneous intervention, and ejection fraction $\leq 40\%$.

‡For a list of covariates, see Table 2 footnote (†).

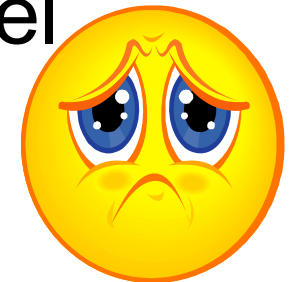
You need to include only propensity score in the model.

Control of confounding with regression model

- Compared to **stratified analysis**, several confounding variables can be easily controlled simultaneously using a multivariable regression model.



- Results from the regression model are readily susceptible to bias if the model is not a good fit to the data.



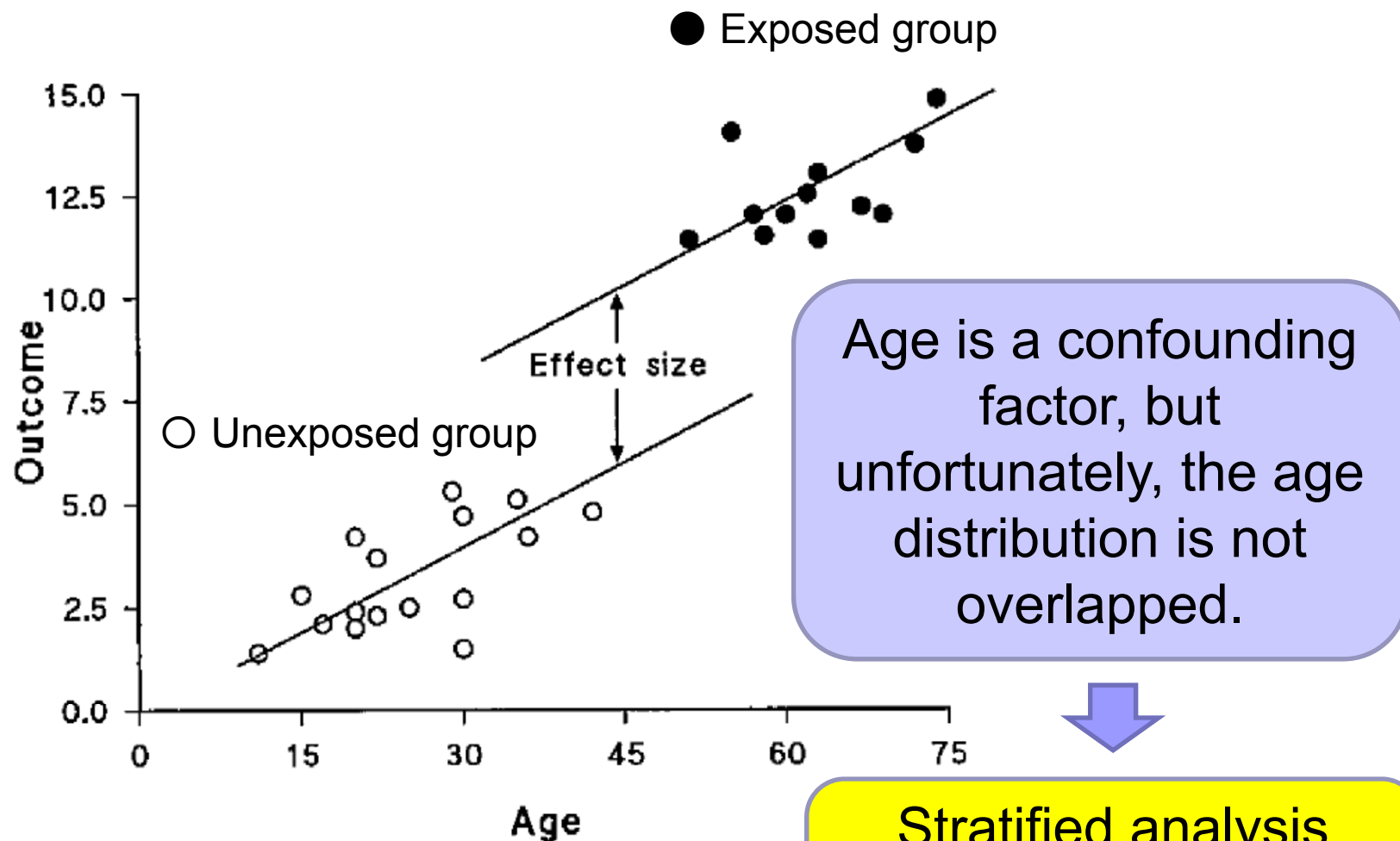
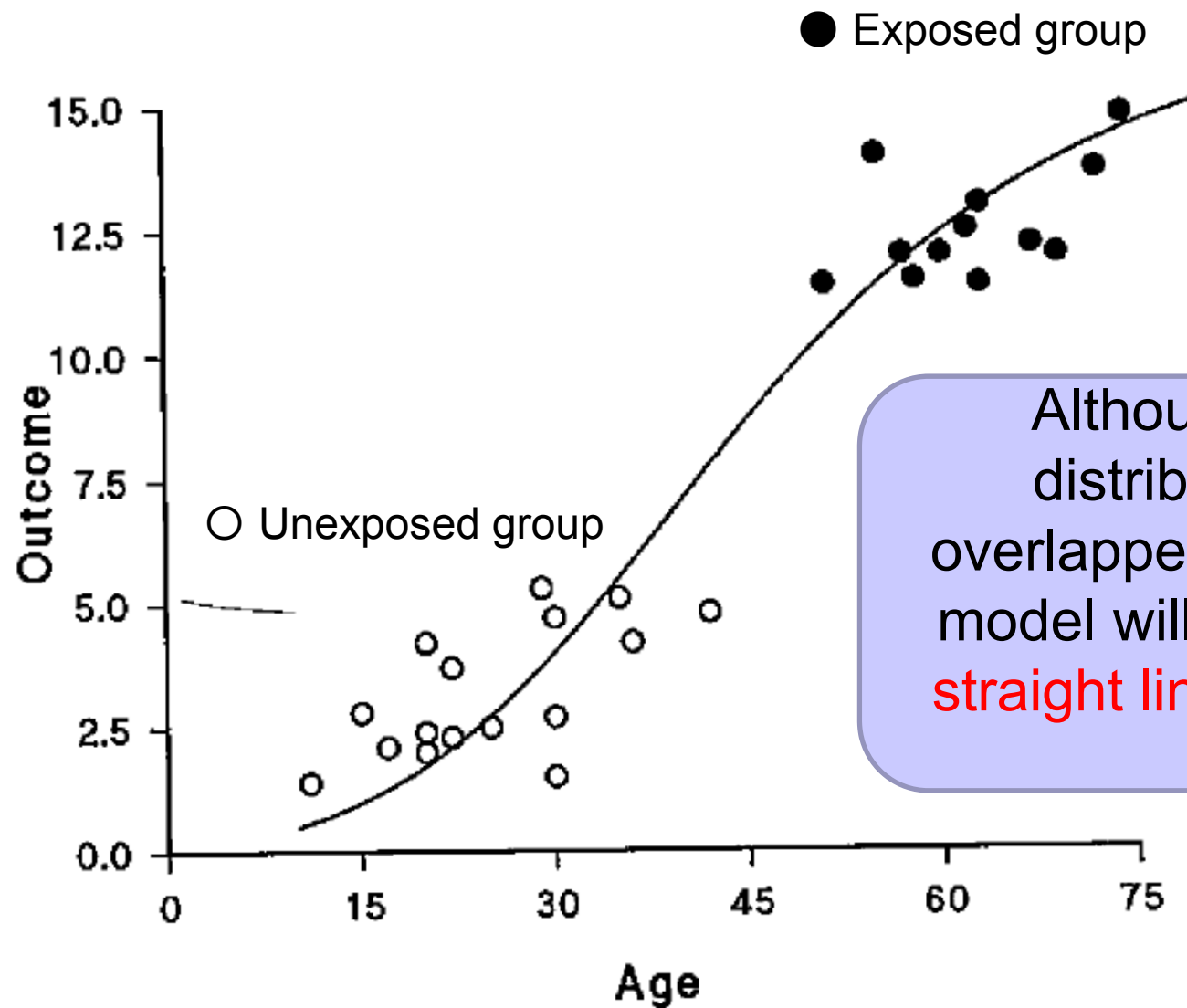


Figure 12-4 Hypothetical example of a multivariable linear regression data involving a dichotomous exposure variable (exposed = solid circles, unexposed = open circles) and age.



Although the age distribution is not overlapped, a regression model will fit **two parallel straight lines** through the data.



Statistic significance vs. Clinical significance

Statistic significance $\neq$ Clinical significance

- P value(s) do NOT tell us the significance in clinical practice / biological importance.
- If your sample size is quite large, you may obtain a result with statistic significance.
So what?

RCT of donepezil for Alzheimer's disease

Lancet. 2004 Jun 26;363(9427):2105-15.

Long-term donepezil treatment in 565 patients with Alzheimer's disease (AD2000): randomised double-blind trial.

Courtney CJ, Farrell D, Gray R, Hills P, Lynch J, Sallwood E, Edwards S, Hardman W, Raftery J, Crome P, Lander C, Shaw H, Bentham P; AD2000 Collaborative Group.

⊕ Auth

Abstract

BACKG

determin

psycholo

METHOD

allocated donepezil (5 mg/

double-blind treatment co

defined by loss of either

assessments were sought

FINDINGS: Cognition averaged 0.8 MMSE

points better (0.5-1.6; p<0.0001) with

institutionalisation (42% vs 44% at 3 ye

care in the donepezil group compared

institutional care was 0.96 (95% CI 0.7

and psychological symptoms, carer psy

donepezil.

INTERPRETATION: Donepezil is not cost effective, with benefits below minimally relevant thresholds. More effective treatments than cholinesterase inhibitors are needed for Alzheimer's disease.

Cognition averaged 0.8 MMSE points better (95%CI 0.5-1.2; p<0.0001) and functionality 1.0 BADLS points (0.5-1.6; p<0.0001) with donepezil over the first 2 years.

Donepezil is **not** cost effective, with benefits below minimally relevant thresholds. More effective treatments than cholinesterase inhibitors are needed for AD.